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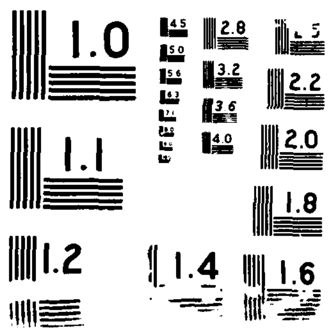
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A Review of the Propagation of Pressure Pulses Produced by Small Underwater Explosive Charges

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This report reviews, from the point of view of nonlinear acoustics, the propagation of pressure pulses due to small underwater explosions, such as those used in scattering experiments. We have concentrated our attention on the peak-pressure region, as it is here that nonlinear effects are likely to appear. It has been found that for some values of the physical parameters, there are ranges where the nonlinear theory gives values for the peak pressure that are quite close to those predicted by the experimental, $(W^{1/3}/R)^{1.13}$ fit. This agreement is found for relatively strong pressure pulses. However, it is concluded that for typical charges used in scattering experiments, nonlinear effects must be negligible, except very near the charges. By implication, it is concluded that except for very close ranges where the shock front is too strong to be considered as a weak shock wave, the decay of the pressure peak in such pulses should be as predicted by linear acoustic theory. As this is not found experimentally, it is suggested that some effects may have been present in the experiments that are not represented by the empirical fit.

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CONTENTS

MAIN FEATURES OF PRESSURE PROFILE

In this section, we describe some of the main features of the phenomena under consideration, and give a brief description of the basic mechanisms that are responsible for the observed pressure profiles. Both of these descriptions are given here simply to introduce the physical quantities that are used later, and to set the stage for subsequent discussion. More complete descriptions can be found in the open literature.

The basic purpose of the experiments is to measure the backscattering, produced by the ocean surface, of acoustic signals generated by explosions of charges detonated at a depth of approximately 1500 ft (Fig. 1).

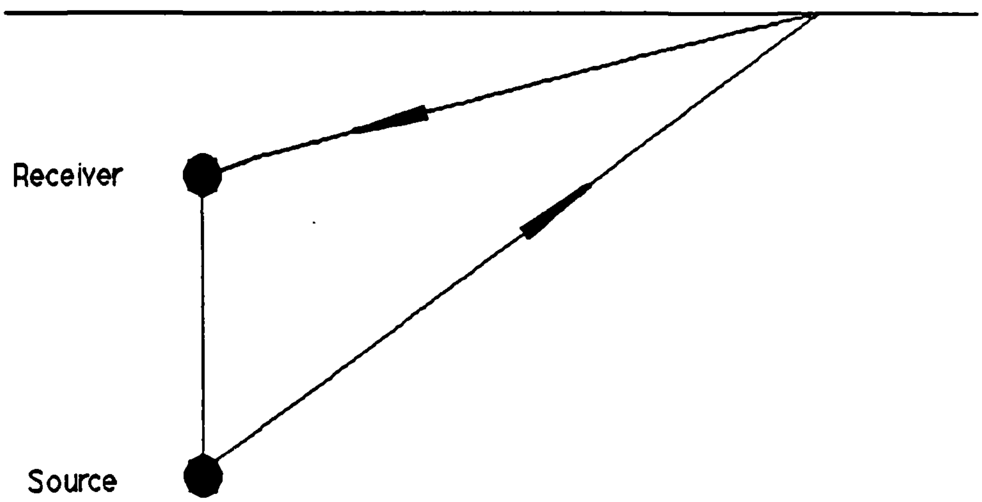

Fig. 1. Scattering-experiment geometry

This review does not consider the scattered signal; it is limited to a consideration of some mechanisms that might play a role in the propagation of the direct pulse. A proper understanding of this propagation is necessary as the amplitude levels in the direct signal affect the estimated source level strengths, and this, in turn, affects the backscattering strength through the sonar equation.

A REVIEW OF THE PROPAGATION OF PRESSURE PULSES PRODUCED BY SMALL UNDERWATER EXPLOSIVE CHARGES

INTRODUCTION

Acoustic pulses produced by the explosion of small charges are used for a variety of purposes in ocean acoustics. As is well known, these pulses have very sharp leading fronts, even for the smallest charges commonly used. These sharp fronts are the result of a balance between dissipation mechanisms, which tend to smooth wave profiles, and nonlinear effects, which tend to steepen them. Because these shock wave fronts are observed at long distances from the explosion, and because even at these distances the amplitudes of the pulses do not seem to decay as predicted by linear acoustic theory, it has sometimes been assumed that nonlinear effects are present which may affect the propagation and reflection of the pulses.

This review considers, from the point of nonlinear acoustics, some of the main features of the propagation, with the purpose of determining whether nonlinear mechanisms affect those features. One of the main conclusions of the review is that for the typical charges used in scattering experiments, nonlinear effects are not important, except very near the charge producing the pulse. In addition, several other mechanisms are mentioned which could play a role in the propagation.

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Because the scattered signal arrives at the location of the receiver over durations that are measured in seconds, experimental scattering measurements are usually made over durations which are too large to resolve the short duration profiles produced by the explosion of the charge, as the main features in those profiles take place in a few milliseconds. These profiles, as well as the corresponding energy spectra, can nevertheless be obtained from similarity curves available in the literature which were obtained from a large number of experiments conducted in the last 40 years. The existing data appear to provide a good quantitative description of those profiles in terms of the sequence of events taking place after the charge is exploded. By this, we mean the very rapid conversion of the solid material, of which the charge is made, into a gaseous mass, commonly referred to as the bubble, at very high temperature and pressure. The conversion is, of course, a rather complicated chemical phenomenon, but its details are not of primary importance in this discussion. What is presently relevant is that the conversion is caused by a detonation wave travelling in the charge at a very large speed. For TNT, for example, this detonation speed is of the order of 20,000 ft/sec. Thus, a one foot radius sphere of explosive would be converted into an incandescent gas at a very high temperature and pressure in less than 25 μ sec. For many purposes, this conversion can be therefore assumed to take place instantaneously, so that at time $t=0$, say, we have a region of high pressure surrounded by water at considerably lower pressures. For example, the detonation of a 3 lb charge of TNT produces, at the surface of the hot bubble, a pressure which is of the order of 20×10^9 dyn/cm^2. If the charge had been detonated at a depth of 600 m, the hydrostatic pressure outside the charge would be about 6.2×10^7 dyn/cm^2, or about 300 times smaller. Such pressure differences are, of course, statically untenable, and a variety of processes are set in motion that tend to eliminate them, including a substantial expansion of the bubble. Of principal importance to this report are the pressure waves that are produced as a result of this expansion. Appendix A gives a heuristic description of these waves, in terms of the instantaneous motion of the bubble. This description is intended to give some insight about the main features of the the measured profiles, which typically appear as the one shown in Fig. 2 below. The circles under the profile give

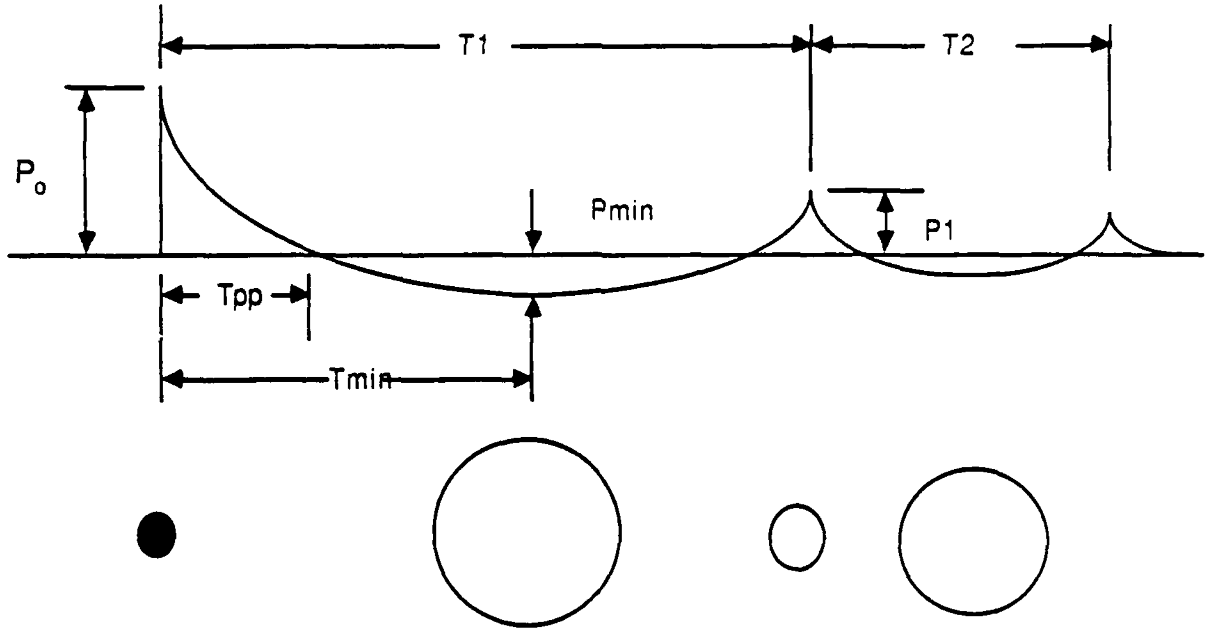

Fig. 2 Pressure profile not too near the charge.

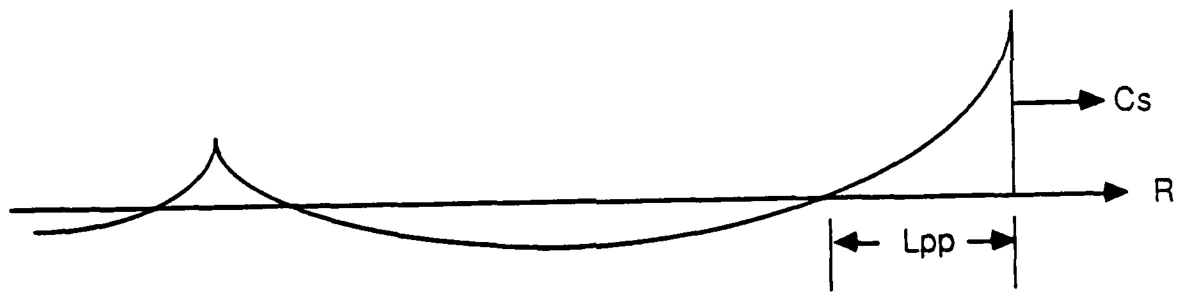

Fig. 3 Radial distribution of overpressure at a fixed time.

an idea of the relative size of the bubble at the moment that a given feature in the profile was produced.

The pulse can be also depicted as a traveling wave by assuming that the pulse is a radially outgoing spherical wave, with features depending on the distance *R* and on time *t* through the variable (R - ct), where c is the speed of propagation. Thus, at some instant of time, the distribution of pressure along the radial direction may have the following appearance.

To fix ideas, we give in the table below approximate values of the relevant quantities for a pulse produced by a 1.8 lb TNT charge, burst at a depth of 800 ft, and measured at a radial distance of 300 ft.

Table 1. Approximate Values of Relevant Parameters for a 1.8 lb TNT Charge.

Symbol	Quantity	Value
R	Distance between charge and shock front	300 ft
W	Weight of explosive charge	1.8 lb
P_h	Hydrostatic pressure at 800 ft	150 lb in^{-2}
P_o	Peak Pressure	61 lb in^{-2}
P_{min}	Minimum underpressure	22 lb in^{-2}
L_{pp}	Length of first positive-pressure phase	7.7 m
T_{pp}	Duration of first positive-pressure phase	5.3 msec
T_1	Time between shock and first bubble pulse	42 msec
T_2	Time between first and second bubble pulse	29 msec
T_3	Time between second and third bubble pulse	24 msec

The values given in the table were obtained from empirical fits of experimental data that have been obtained over the last 40 years by several investigators, and which are believed to be accurate representations of the respective parameters in actual pulses. Of central importance to much of the work with explosive sounds are the fits for the peak pressure P_o, and for the duration of the various phases of the pulse. The fits for the peak and first bubble pulse pressures, as given by Slifko (1967), are

$$P_o = 2.08 \times 10^4 \, (W^{1/3}/R)^{1.13} \quad \text{psi}$$

$$P_1 = 3300 \, W^{1/3}/R \ \text{psi}$$

where W is in pounds and R is in feet. Fits for other quantities will be considered later. A more complete list of them is given in Appendix B.

The dependence of these pressures on the ratio $(W^{1/3}/R)$ is of considerable importance in the determination of the energy spectrum levels for the complete pulse. It should be noted that in the peak pressure fit, that ratio appears raised to the 1.13 power, whereas in linear acoustic theory, it is raised to the first power. At a range of few hundred meters, for example, such differences in the exponent can give rise to a few dB difference in the energy spectrum levels.

Historically, the $W^{1/3}/R$ dependence is known as *Hilliar's principle,* which for spherical charges may be expressed as follows. "Distances to equal overpressures, are proportional to the cube roots of charge weights." By 1924, Wood had found that the principle did not hold well for the peak pressure produced in shallow underwater explosions. Instead of the $W^{1/3}/R$ dependence, he found that the peak pressure scaled as

$$W^{0.38}/R = (W^{1/3})^{1.13}/R$$

Although Wood could not explain the differences, he noted that the principle did not include many factors present in the experiments. We note in passing that the dependence of Wood's peak pressures on distance did scale with the predicted 1/R, and that this scaling coincides with acoustic theory. Nevertheless, in 1946, Coles *et al* reported pressure peak results that were well fitted by

$$P_o = 2.16 \times 10^4 \, (W^{1/3}/R)^{1.13} \quad \text{psi}; \quad 0.03 < W^{1/3}/R < 0.85 \quad lb^{1/3} ft^{-1}$$

Because the raw data upon which this result is based is still available, we may use the data to show some of the features of the correlation. The data were obtained with charges weighing 48 and 76 pounds, burst at a depth of 40 ft. The table below shows the 76 lb data. The 48 lb data shows similar trends.

Table 2. Pulse data obtained by Coles *et al.* W = 76 lbs.

Shot No.	R, ft	P_o, psia	ϑ, msec	$W^{1/3}/R$, $lb^{1/3}/ft$
393	5	18950	0.260	0.847
387	7	12400	0.270	0.605
385	10	8185	0.300	0.424
393	15	4530	0.335	0.282
387	17	4215	0.315	0.249
385	20	3755	0.325	0.212
393	38	2115	0.355	0.111
387	47	1595	0.355	0.090
385	60	1066	0.435	0.071
393	78	814	0.435	0.054
387	87	760	0.440	0.049
385	100	760	0.480	0.042

The quantity ϑ given in the fourth column is the elapsed time since the arrival of the shock front and the place on the wave where the pressure

equals 1/e of the peak value. ϑ is therefore a measure of the duration of the positive-pressure region in the pulse. The last column, giving $W^{1/3}/R$ was added for convenience, as it gives the value of the variable used to fit the data.

The correlation was given further support when in 1954 Arons published additional shallow-water data that extended it to lower values of $W^{1/3}/R$. Thus, for TNT charges, he found that

$$P_0 = 2.16 \times 10^4 \, (W^{1/3}/R)^{1.13} \text{ psi}; \qquad 0.005 < W^{1/3}/R < 0.85 \text{ lb}^{1/3}\text{ft}^{-1}$$

Incidentally, the pressure differences predicted by the constant of proportionality in the fit changing from 2.08×10^4 to 2.16×10^4 is well within the experimental errors. In our computations we will use the later figure.

While Arons's paper does not contain a detailed description of his experiments, he did note that they were performed in well mixed tidal waters, and that refraction effects were known to be *negligible*.

Arons also attempted to compare his results for large ranges with those predicted by the asymptotic theory of Kirkwood and Bethe. This theory predicts that for large ranges, the peak pressure should decay with distance as

$$P_0 \sim (R_0/R)\,(\ln R/R_0)^{-1/2}$$

where R_0 is a reference distance (Arons's paper has a typographical error in this equation, which appears there without the 1/2 power on the logarithmic term). What Arons attempted to do was to fit the logarithmic term, in a given range of $W^{1/3}/R$, with a power function. While he found this could be done, at least in a limited range, he also found that the power function had a slope of 0.06 rather than the experimentally determined value of 1.13. An implication of this is that the asymptotic prediction of the Kirkwood-Bethe theory is in question. As we will see

later, such a conclusion is not warranted. Nevertheless, it should be emphasized that the theory does not predict a decay proportional to $(W^{1/3}/R)^{1.13}$ as stated by some authorities.

Extension of the correlation to deep water explosions was first reported by Blaik and Christian, who in 1965 published measurements of pulses from charges at depths varying from 3000 to 22000 ft. The measurements were made directly above the charges to avoid refraction effects. While some differences were found relative to the shallow-water data, the peak pressure fit was found to agree with Arons's, thus extending its range of applicability. Thus,

$$P_o = 2.16\times10^4\ (W^{1/3}/R)^{1.13}\ \text{psi}; \qquad 2.5\times10^{-5} < W^{1/3}/R < 0.85\ \text{lb}^{1/3}\text{ft}^{-1}$$

Additional work in both shallow and deep water was reported in 1967 by Slifko, who performed measurements for bursts at depths ranging from 500 to 14,000 ft. Slifko's measurements were also performed directly above the charges. His report also contains valuable fits for all quantities of importance in the pulse, as functions of depth, weight and range. For the case of the peak-pressure of pulses due to TNT charges, Slifko's results are within 4% of those previously reported, viz.,

$$P_o = 2.08\times10^4\ (W^{1/3}/R)^{1.13}\ \text{psi}; \qquad 7.1\times10^{-5} < W^{1/3}/R < 0.85\ \text{lb}^{1/3}\text{ft}^{-1}$$

If we disregard the small differences between the several fits, then it follows that the pressure-peak correlation is accurate for values of $W^{1/3}/R$ ranging from 7×10^{-5} to 8.5×10^{-1}. This implies, for example, that for a 1 lb charge, the correlation correctly predicts the peak pressure in pulses as close as 1.18 ft to the center of the charge, and as far as 40,000 ft from it.

A remarkable conclusion of all this is that even at such long ranges as those given by the upper range just computed, does the pressure peak decay as $R^{-1.13}$, rather than the theoretical acoustic decay given by R^{-1}. The most obvious implication is that for *all* values of R, the

pressure decays more rapidly than linear theory predicts, as indicated by Fig. 4, where the two decays are plotted as a function of (R/R_o).

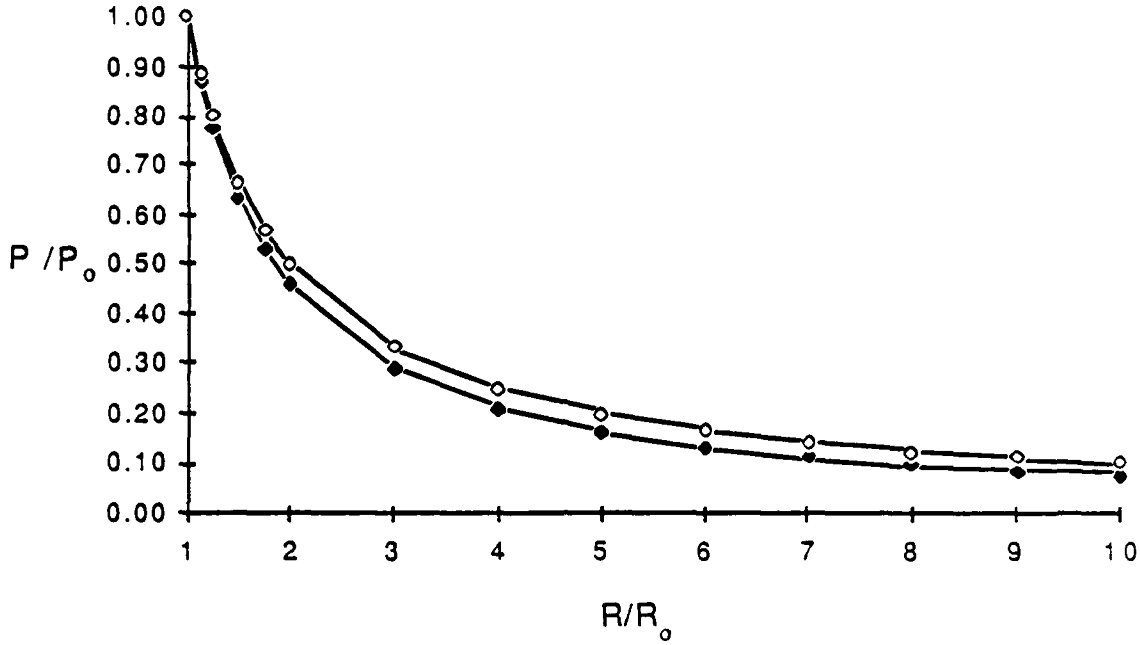

Fig. 4. Linear and empirical pressure peak decays

POSSIBLE NONLINEAR EFFECTS

Because the front of the wave is a shock wave, it has often been suggested that the discrepancies between observation and acoustic theory are due to the nonlinear effects that maintain the sharp front. This possibility is considered here in terms of the simplest possible model; e.g., a spherical pulse expanding in an infinite, homogeneous medium devoid of viscosity. This model is only intended to provide an order of magnitude estimate for the effects of nonlinearity. We will limit this discussion to the nonlinear effects that may take place between the front of the wave and the first point in the pulse where the overpressure vanishes. This region is depicted below for pulses that are triangular in shape.

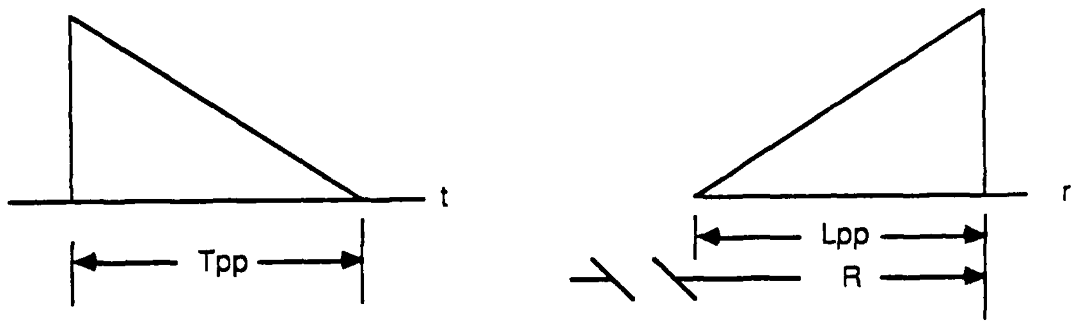

Fig. 5. Front region of pressure profile

In an actual pulse, the pressure decay regions are not straight lines as shown above, but for the purposes of this discussion, these differences can be disregarded.

It should be added that considering only the first positive region of the total pulse is sufficient, as this portion of the pulse is effectively decoupled from the rest. The reason for this is simply that the point of zero pressure has, at sufficient distances from the charge, also zero fluid velocity. Thus, that point propagates with exactly the ambient speed of sound. No wavelets on any portion of the profile can propagate through this point because (see Fig. 3) to its left the speed of propagation is lower, so that wavelets in that region move to the left. Similarly, wavelets to the right of the zero crossing point move faster than the ambient speed of sound, and therefore move to the right of that point.

Now, every experimental investigation with explosive sounds in water shows that near the charge the shock wave is followed by an exponential decay, with no apparent zero crossing point. This is the strong shock wave region. Further out, the profile develops near its front a nearly triangular region which is maintained for considerable distances. This is the region where the weak shock wave theory applies. In this region, changes may be taking place in the pulse front, but the agencies causing them, e.g. dissipation, are apparently not able to eliminate the rapid transition, or shock, at the front. Nevertheless, a significant amount of energy dissipation must take place there, for dissipation due to viscous and thermal effects is proportional to the squares of the velocity and

temperature gradients, and these are necessarily large at the shock. Thus, one effect of the sharp front is to produce dissipation of the energy of the pulse, and this should result in a faster decrease of the pulse's amplitude than that due to wavefront spreading alone.

A second effect, solely due to nonlinearities, is that the length (or duration) of the nearly triangular pulse increases as the pulse travels. This is caused by points in the shock front travelling with a speed that is larger than the speed of the zero-crossing point.

The combined results of lengthening and attenuation due to dissipation at the shock may be visualized in terms of two snapshots of the pulse taken at two different instants. If we now take the zero-crossing points of both and superimpose them, we get profiles similar to the following figure.

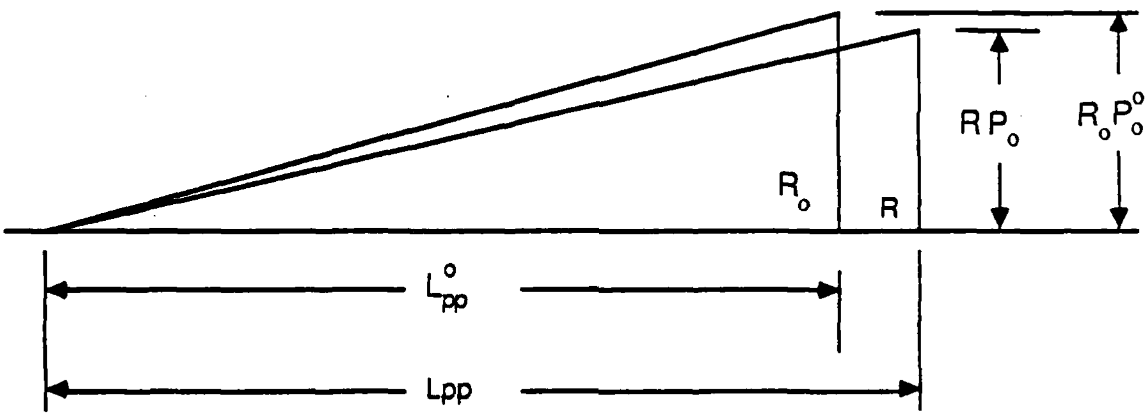

Fig. 6. Superimposed pressure profiles

In this figure, the superscript zero on P_o^o represents the peak pressure evaluated at some fixed distance R_o.

The theory describing such effects is well developed (see, for example, DuMond *et al*, 1946; Landau and Lifshitz, 1959; Blackstock, 1972), and has been experimentally verified for both short spherical pulses (Wright, 1983) and for plane pulses (Temkin and Maxham, 1985). In the notation of Fig. 4, the spherical-case theory gives for spherical waves at distances R large compared with the length L_{pp} of the pulse

$$P_o = P_o^o \frac{R_o/R}{\sqrt{1 + \beta_o\,(P_o^o/\rho_o c_o^{\,2})\,(R_o/L_{pp}^o)\,\ln\,(R/R_o)}} \qquad (1)$$

$$L_{pp} = L_{pp}^o \sqrt{1 + \beta_o\,(P_o^o/\rho_o c_o^{\,2})\,(R_o/L_{pp}^o)\,\ln\,(R/R_o)} \qquad (2)$$

In these equations ρ_o is the ambient density and β_o is a thermodynamic property of the fluid, known in the acoustic literature as the coefficient of nonlinearity. It is given by

$$\beta_o = 1 + \rho_o^{\,3} c_o^{\,4} \left(\frac{\partial^2 \rho^{-1}}{\partial p^2}\right)_{s,\rho = \rho_o} \qquad (3)$$

where s is the entropy per unit mass of fluid. The quantity β_o can be written in other useful forms, including

$$\beta_o = 1 + \rho_o c_o \left(\frac{\partial c}{\partial p}\right)_{s,\rho = \rho_o} \qquad (4)$$

$$= 1 + \frac{1}{2}\frac{B}{A}$$

The last form gives β_o in terms of the nonlinearity parameter (B/A), which plays an important role in nonlinear acoustics, particularly for fluids whose equations of state are known only experimentally. Thus, while the value of β_o can be computed for a perfect gas from the perfect gas equation of state, its value for other fluids must be determined experimentally. For sea water at 20 C and at atmospheric pressure, for example, experiments give (see Beyer, 1974)

$$B/A = 5.25$$

Landau (1945) appears to have considered nonlinear effects first (although Brinkley and Kirkwood made similar calculations independently). These investigators correctly argued that for sufficiently large values of R, the second term in the radical of Eq. (1) or (2) becomes dominant, so that at large distances from the explosion, the peak pressure should decrease as

$$P_o \sim (R_o/R)\ (\ln R/R_o)^{-1/2}$$

This is the asymptotic expression used by Arons, who, as noted earlier, found that it does not compare well with the results of measurements at long distances. As we will show later, the asymptotic expression does not apply for the relatively weak shocks used in underwater acoustics research.

Equations 1 and 2 are used as follows: Select an initial distance to the shock front where the theory is expected to be accurate, i.e., where the scaled pressure amplitude $P_o^0/\rho_o c_o^{\ 2}$ is small, and where R_o/L_{pp}^o is large.

The nondimensional pressure amplitudes and lengths at other ranges R are the obtained by simple substitutions of these parameters into Eqs. 1 and 2. Figure 7 shows theoretical predictions for three values of the parameter

$$\tau = \beta_o (P_o^0/\rho_o c_o^{\ 2})\ (R_o/L_{pp}^o)$$

The case $\tau = 0$ corresponds to linear, ideal-fluid theory. It is thus seen that the nonlinear decay, which includes dissipation at the shock front, is faster than that predicted by linear, ideal acoustics. Had dissipation in the main body of the wave been taken into account, the decay would be faster for both linear and nonlinear decays, with the nonlinear theory still predicting a more rapid decay than the linear theory. The reason for this is that, for waves with weak shocks, the region outside the shock, the wave behaves linearly. That is, in this approximation, the energy dissipated in the main portion of the wave can be computed from linear theory. This energy dissipation is thus the same for both linear and nonlinear waves.

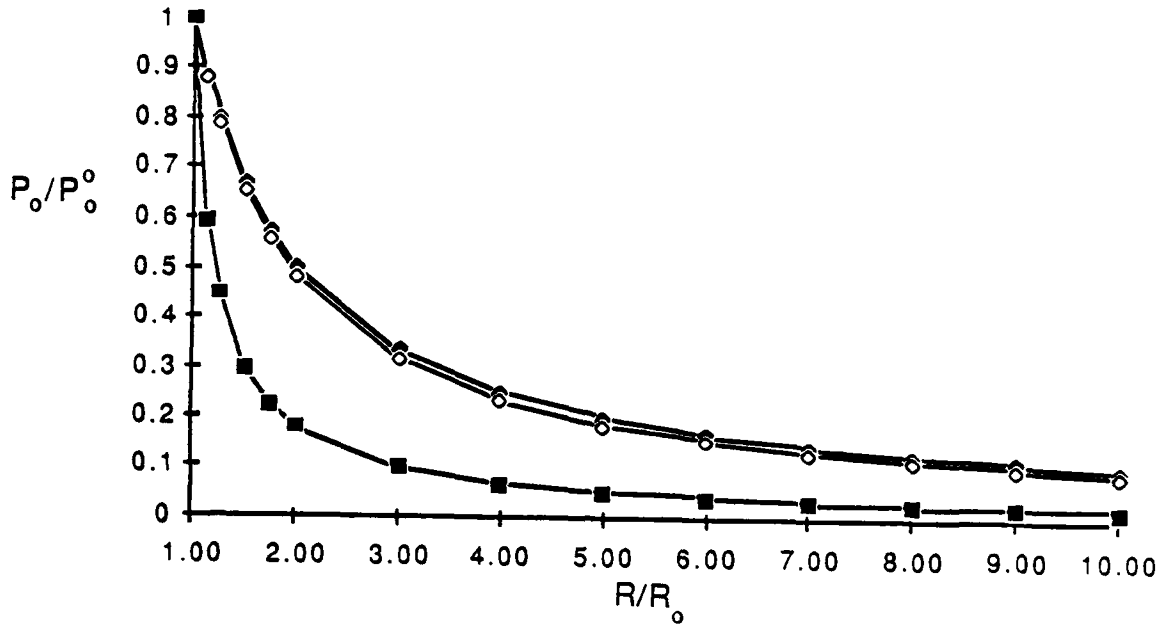

Figure 7. Nonlinear decay. ♦: τ=0 ; ◊: τ=0.01; ■ : τ=10.

To compare the nonlinear decay prediction for the peak pressure with the empirical fit, we express the fit in a non-dimensional form as

$$(P_o/P_o^o)_{fit} = (R_o/R)^{1.13}$$

As this form shows, the fit, like the ideal theory, also predicts a non-dimensional decay which depend only on the non-dimensional range (R_o/R). By contrast, the nonlinear decay depends on initial amplitude through the parameter τ in equation (1). Figure (8) shows graphical comparisons between nonlinear theory and fit for two values of τ used in Fig. 8. Again, the curve with τ equal to zero corresponds to the ideal theory.

The other curve is merely intended to show the results of a different value of τ . As one of the two values of τ predicts slower decay than the fit, whereas the other predicts a faster decay, it ought to be possible to obtain a value of τ which will match the fit, at least for some range of R/R_o. This is, of course, what Arons attempted to do. He however, used the asymptotic expression given earlier. A cursory inspection of Eq. (1) shows that for realistic values of τ, the asymptotic expression does not apply, except at very large values of R/R_o, because the logarithmic function

increases very slowly. Thus, a one hundred fold increase of R/R_o increases $\ln(R/R_o)$ only by a factor of 2. It is therefore clear that in order to find a match between the two predictions, both terms in Eq. (1) have to be retained. That this is indeed the case can be seen in Fig. 9, which shows the two decays for $\tau = 0.37$, in the range $1 < R/R_o < 10$. The trends begin to diverge at values of R/R_o equal to about 100, as shown in Fig. 10.

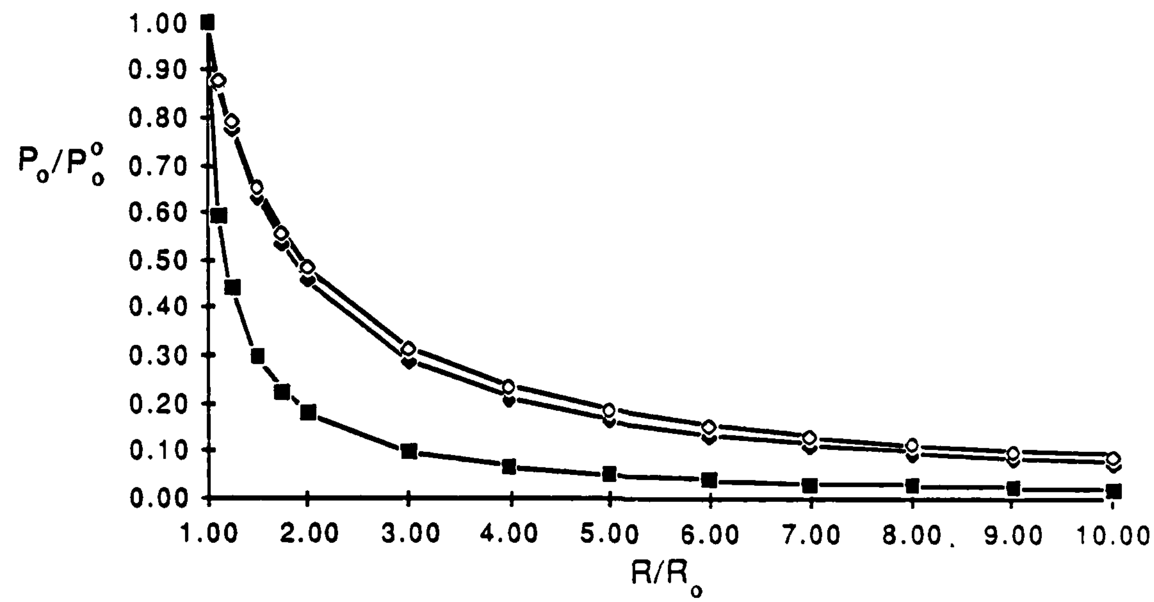

Fig. 8. Empirical fit and nonlinear decays. ◊: τ=0 . ♦: Empirical fit .■: τ= 10.

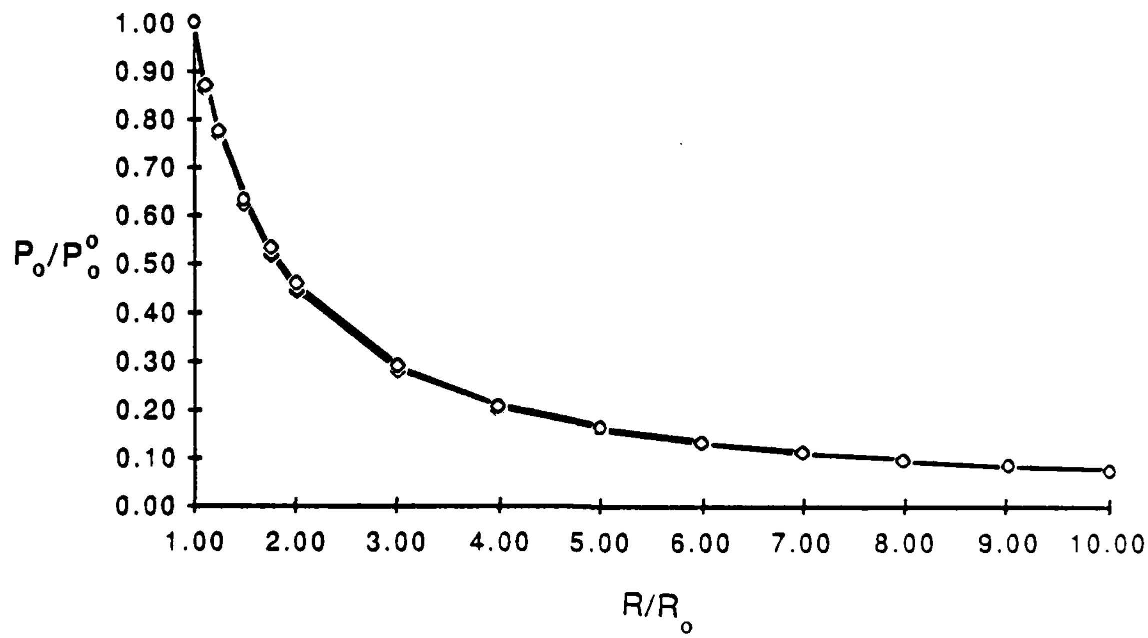

Fig. 9. Empirical and nonlinear pressure-peak decays. τ=0.37.

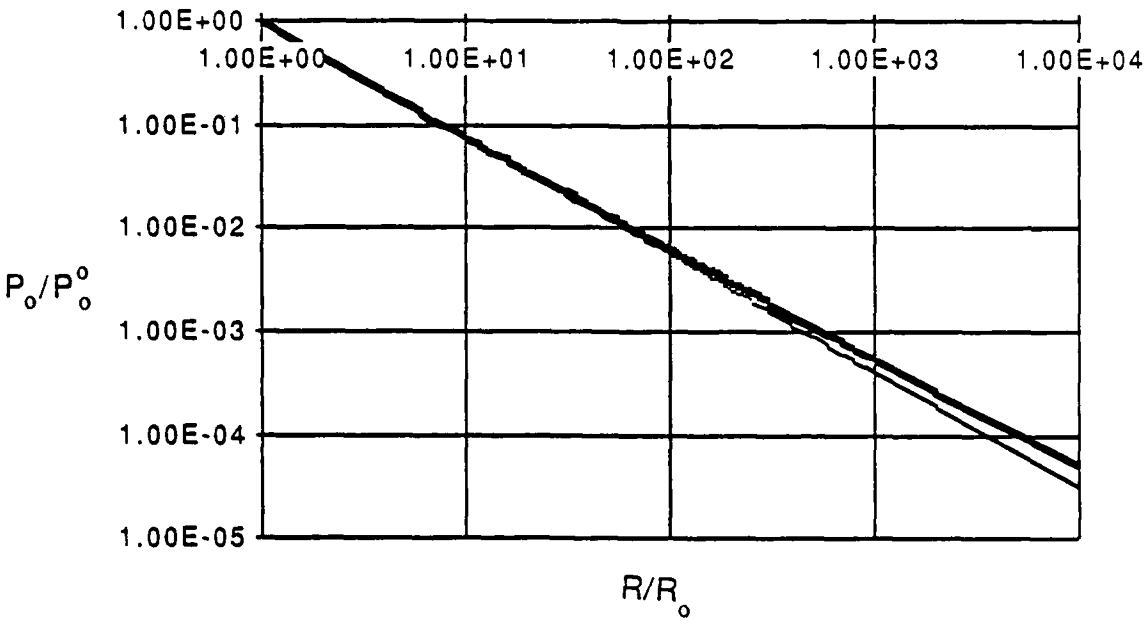

Fig. 10. Empirical and nonlinear pressure-peak decays. Thick line represents nonlinear theory for τ=0.37.

While the agreement between the theoretical curve for τ =0.37 and the fit appears to be reasonable, it should be remembered that we forced the agreement by selecting a value of τ that gave good results. For this agreement to be meaningful, the chosen value of τ must be one that applies to an actual case, and the particular values of $P_o^o/\rho_o c_o^2$ and R_o/L_{pp}^o must be such that they fall within the range of applicability of the nonlinear theory. For this theory to be applicable, the first quantity must be small, and the distance R_o should be large compared to the initial length L_{pp}^o at that location.

To compare the fit and nonlinear theories in a particular example, we need to obtain values of $P_o^o/\rho_o c_o^2$ and R_o/L_{pp}^o at some range R, due to the explosion of a given charge. To obtain these parameters, we use the empirical fits presented earlier. Thus, the length L_{pp}^o can be estimated from the fit for T_{pp} by means of

$$T_{pp}/T^{o}_{pp} = L_{pp}/L^{o}_{pp} \qquad (5)$$

This equality holds because the length L_{pp} is related to the duration T_{pp} by means of $L_{pp} = c_o T_{pp}$.

As a "typical" case of interest to some scattering experiments, we consider the pulse produced by an 1.8 lb TNT charge detonated at a depth of 1000 ft. According to the correlation given by Slifko, the duration of the first positive phase is given by

$$T_{pp} = 0.555\ W^{1/3}/z^{5/6}$$

If we accept this result, which states that T_{pp} is independent of range, so that no lengthening takes place, then $T_{pp} = 2.08$ msec. Using a nominal speed of sound of 1450 m/sec, this gives a length $L_{pp}{}^{o}$ of about 10 ft. Thus, if we take $R_o = 50$ ft, the ratio R_o/L^{o}_{pp} is approximately 5, and may be therefore taken as "large." At this distance, the peak pressure in the pulse is $P_o = 2.16\text{x}104\ (1.8^{1/3}/50)^{1.13} = 324$ psi. While this is a significant absolute overpressure, it is nevertheless very small when compared with $\rho_o c_o{}^2$. For sea water, the nominal value of this quantity is 2.25×10^{10} dyn/cm^2, or 2.25×10^4 Atm. Hence, for this example, $P^{o}_{o}/\rho_o c_o{}^2 \approx 1\text{x}10^{-3}$. The last parameter that remains to be evaluated for the example is β_o. We saw earlier that at 20° C and at atmospheric pressure, it is equal to about 4. Morfey (1984) computes the variation of that quantity with pressure and salinity, and gives values that are about 4-5. For simplicity, we will simply assume that it does not exceed the value of 5 throughout the range of interest. Thus, the decay prediction including nonlinear and spreading effects gives for this numerical example

$$P_o/P^{o}_{o} = (R_o/R)\ \{1 + 5\ \text{x}10^{-2}\ \ln R/R_o\}^{-1/2}$$

At a distance of 500 ft, or 10 times R_o, this gives 0.947 (R_o/R). At 5000 ft, it gives 0.902 (R_o/R). For comparison, the decay predicted by linear

theory is simply given by the wavefront spread factor (R_o/R), whereas that predicted by the empirical fit, can be expressed as

$$(P_o/P_o^o)_{fit} = (R_o/R)^{1+.13}$$

At $R/R_o = 100$ this gives 0.55 (R_o/R), representing a considerably faster decay than either the linear or the nonlinear decay.

We have thus shown that for small charges, nonlinear effects produce a few percentage difference in the peak-pressure values at moderate values of (R/R_o), and at distances from the charge which are not too small. This conclusion follows simply from the smallness of the quantity $P_o^o/\rho_o c_o^2$ in the selected example, and applies to other situations provided the main assumptions are satisfied. These are as follows:

1. Refraction effects can be disregarded.
2. Curvature effects are negligible.
3. Dissipation exists only at the shock front.
4. Nondimensional pressure amplitudes $P_o^o/\rho_o c_o^2$ are small.

As described later, some of these assumptions require further investigation.

We may turn the question around and ask when are nonlinear effects so significant that the decays predicted by the nonlinear theory significantly depart from linear theory. This occurs, of course, when the quantity multiplying the logarithmic term in Eq. (1) is large. However, such large values of τ are outside the limits of applicability of the nonlinear theory. Nevertheless, if τ is of order one, nonlinear effects are significant, as the agreement with the experimental fit for τ equal to about 0.37, indicated. Also, for $\tau = O(1)$, it is possible to satisfy the basic assumptions of the theory. We will therefore use a value of $\tau = 1/2$ to guide the discussion. This relatively large value of τ could easily occur if the ratio $P_o^o/\rho_o c_o^2$ is

sufficiently large. It should nevertheless be remembered that weak shock theory requires that quantity to be small. One reason for the limitation is that weak shock theory depends on an approximation for the entropy increase across the shock which is valid only for small pressure differences across it. In air, for example, weak shock theory predicts entropy increases across a shock wave which agree within 15% of the actual increases for values of $P_o^o/\rho_o c_o^2$ up to about 0.1 (Temkin, 1969). No similar comparisons seem to have been made for shocks in water. Nevertheless, we must take $P_o^o/\rho_o c_o^2$ to be "small." *The quantity τ multiplying the logarithmic term could still be of* order one if $P_o^o/\rho_o c_o^2 \approx L_{pp}^o/R_o$, with $R_o/L_{pp}^o >>1$, as required. Taking τ equal to 0.5, as indicated above, we obtain

$$P_o^o/\rho_o c_o^2 \approx 0.5\ [\beta_o(R_o/L_{pp}^o)]^{-1}$$

Before we compare the nonlinear-decay predictions with actual data, let us compute the charges that are needed to provide such significantly nonlinear behavior as the case τ =0.5 requires. For this we need values of β_o and (R_o/L_{pp}^o). Now, the value of β_o is nearly constant and equal to about 5. On the other hand, R_o/L_{pp}^o must be "large" for the theory to be applicable. If we take for it a value of 5 , and also put β_o = 5, as before, the approximate equation above, yields $P_o^o/\rho_o c_o^2 \approx 0.02$. While this value is 20 times larger than that found for the scattering-experiment example above, it is not unusually large. Moderate charges may produce it at relatively small ranges. Let us use the empirical fit to compute the weight of the charge that is needed to produce such a peak pressure at some distance R_o from the charge. Using Eq. (B1) we obtain $W^{1/3}/R_o = 0.350\ \mathrm{lb}^{1/3}\ \mathrm{ft}^{-1}$. This value of $W^{1/3}/R$ is within the range of validity of the empirical fit. At R_o=10 ft, this gives W= 43 lb. At smaller distances, the required weights will, of course, be smaller. However, at values of R_o

comparable to the lateral size of the charge, the applicability of the theory is doubtful, because the existence there of a well defined L_{pp} is questionable. Further, even if a well defined L^o_{pp} could be obtained, the assumption R_o/L^o_{pp} >>1 cannot be satisfied.

As might be expected from the above discussion, the nonlinear theory should predict, for τ = O(1), decays that are comparable with those obtained experimentally, at least in a limited range of the parameters involved. This is likely to occur in regions where the amplitude of the wave is large. For moderate charges, this occurs at distances not too far from the charge. Consider the data of Coles *et al*, that was mentioned earlier. They were obtained with 48 and 76 lb charges at ranges varying between 5 and 100 ft. The charge diameters were 12 and 14 inches, respectively. We may therefore attempt to use those data for comparison purposes. Now, the nonlinear theory is not capable of predicting the magnitude of the pressure produced by a given explosion. All it does is to predict the pressure peak decay from a known value obtained at a given distance. Thus, we use the measured value of P_o at the first distance and equate it to P^o_o in Eq. (1). The resulting decay follows from that equation once τ is known. As before, we take τ = 0.5, even though we have no theoretical reason to do so (but as we will later see, such a value may be inferred from the data as well). The results for the 76 lb charges are shown in tabular form below, and in graphical form in Fig. (11).

We may also compare the various decay trends by plotting the data on a log-log scale, or alternatively, by plotting the logs of the data on a linear scale. Figure (12) shows the logs of the experimental, linear, empirical and nonlinear fits, plotted against the log of $w^{1/3}/R$. The linear decay was added to show the divergence in this range of $w^{1/3}/R$. The other two lines give fits which are equally good.

The agreement is somewhat surprising because of the value of τ was not selected to obtain a best fit. Rather, it was chosen because it seemed to be reasonable. As it turns out, however, the figure was of the correct

Table 3. Experimental, Empirical Fit and Nonlinear Peak Pressure Decays.

R ft	$W^{1/3}/R$ $lb^{1/3}ft^{-1}$	Pressure, psi Experimental 76 lb data	Empirical Fit $21600x(W^{1/3}/R)^{1.13}$	Nonlinear $\tau = 0.5$
5	0.847	18950	17908	18950
7	0.605	12400	12244	12523
10	0.424	8185	8183	8165
15	0.282	4530	5175	5075
17	0.249	4215	4492	4390
20	0.212	3755	3739	3641
38	0.111	2115	1810	1757
47	0.090	1595	1424	1384
60	0.071	1066	1080	1055
78	0.054	814	803	788
87	0.049	760	710	699
100	0.042	577	607	600

order of magnitude. Consider, again, the definition of τ. It is given by

$$\tau = \beta_o (P^o_o/\rho_o c_o^{\,2}) \; (R_o/L^o_{pp})$$

To obtain a numerical value for this, approximately valid for the date of Coles *et al*, we use the data they obtained at R= 47 ft. This point is halfway the range they used. This distance becomes R_o. At that location (see Table 2.), P_o= 1595 lb/ft^2, so that $P^o_o/\rho_o c_o^{\,2}$ = 0.005. The value of L^o_{pp} can be estimated from the value of ϑ at that location. This is also given in the table. Thus, $L^o_{pp} = c_o\,\vartheta$. This gives R_o/L^o_{pp} = 27, which in turn yields τ =0.64.

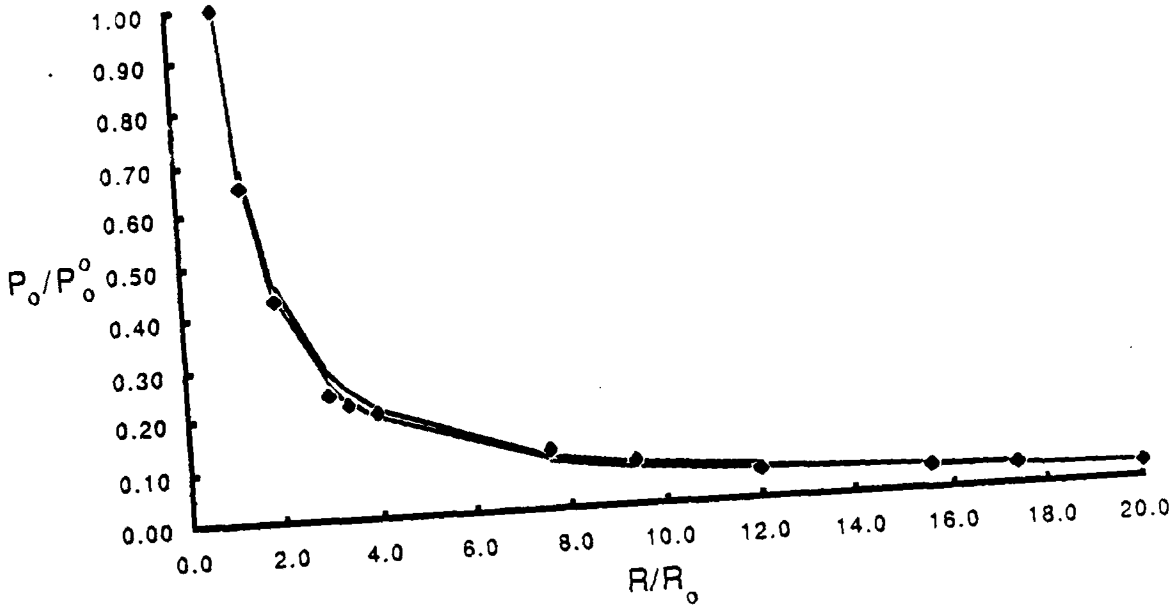

Fig. 11. Pressure peak decay. Solid symbols represent the 76 lb data of Coles et al. The lines represent the $(W^{1/3}/R)^{1.13}$ fit and the nonlinear theory for τ=0.5.

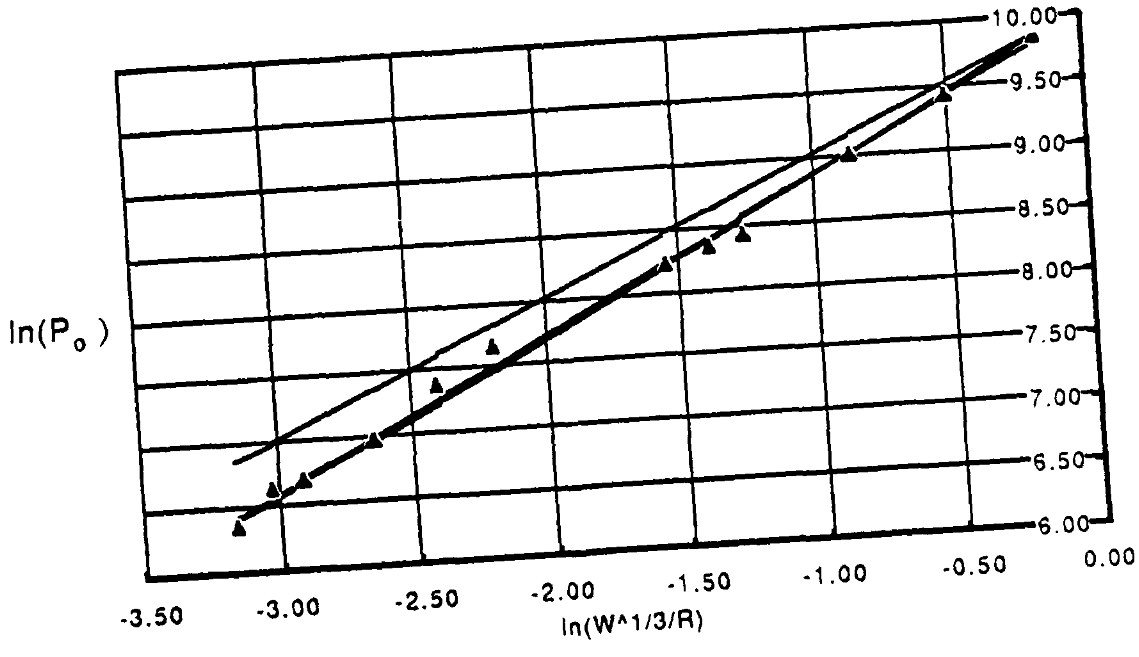

Figure 12. Comparisons of decay trends. 76 lb charges. ▲: Experiments. Upper line is linear theory.

It is thus clear that values of τ can be found which give theoretical predictions in agreement with experimental data. However, for the theory to have any meaning, those values must be realistic for the particular data to which the theory is being compared.

DISCUSSION OF SOME EXPERIMENTAL AND THEORETICAL RESULTS

In section II, we estimated the effect of nonlinearities as they might possibly affect the decay of the peak pressure, over moderate distances, in pulses obtained from underwater explosions of small charges. This portion of the pulse was selected as it is the one most likely to display such effects. Also, other portions of the pulse appear to decay as 1/R, that is, as predicted by linear theory.

Although the evidence for a decay of the pressure proportional to $R^{-1.13}$ is extensive, there are two factors which, in this writers opinion, appear to be inconsistent. These are:

1) For a given range, the peak pressure only depends on $W^{0.38}$

While it is clear that the energy released by a given explosive may scale with its modified lateral size $(W^{1/3})^{1.13}$, the fact remains that the pressure pulse produced by it depends on the ability of the bubble to expand against the surrounding water. The forces resisting the expansion are largely due to the hydrostatic pressure around the bubble, and this depends on the depth. Thus, the pressure peak must depend on depth as well as on range. Investigators have been apparently aware of this point. Blaik and Christian, for example, pointed out that their experimental setup prevented them from separating the effects of range from those of depth. It is also of interest to note that the empirical fit for P_{min} (Eq. B2) does depend on depth, for both shallow and deep explosions.

2) Correlation is not affected by stratification

Except for the initial measurements in shallow waters, which were conducted with either both charge and receiver at the same horizontal levels, or in well mixed waters, the bulk of the pressure-peak data has

been obtained by measuring, near the surface, the pulses produced by charges at depths extending to 22000 ft (Blaik and Christian, Slifko). In order to avoid refraction effects due to stratification, these investigators made the measurements along the vertical, that is, directly above the charge. Such a configuration eliminates bending of the direction of propagation due to stratification. However, as a result of the significant horizontal stratification that must have been present in the experiments between the charge and the receiver, some acoustic energy must have been reflected back to the source. As no calculations of this reflection have been made, the magnitude of the effect cannot be assessed. It is therefore possible that it was negligible. Nevertheless, the significant speed of sound changes below 4-5000 ft, may cause some changes in the amplitude of the wave. Yet, the pressure-peak correlation shows no such effect. In fact, it agrees with data obtained in well mixed waters. Of course, as the gradient of the speed of sound at such depths changes sign somewhere along the vertical propagation, it is also possible that on its way to the surface, a pulse is first amplified due to reflection from layers of decreased impedance, and then attenuated by layers of increasing impedance. It is therefore also possible that reflection and amplification were of about the same magnitude, so as to eliminate stratification effects. None of these possibilities can be assessed without further study of the data and of the effects of stratification on vertical propagation.

Theoretical work

The limitations of theoretical work for nonlinear propagation that was introduced earlier include the effects of: 1). curvature, 2) dissipation in the main parts of the wave, and 3) the effects of stratification.

The first of these effects is likely to be of little importance, as most measurements are taken sufficiently far from the charge, and at these distances the spherical wavefronts may be regarded as being locally plane. Dissipation in the main portion of the wave, may also prove to be of little importance, at least at low frequencies, because that dissipation is proportional to the square of the gradient of the pressure, and this is small everywhere in the main pulse. Further, that dissipation is also

proportional to the kinematic viscosity of the fluid, and for water, this is considerably smaller than for air. However, since these effects affect the peak pressure through an exponential decay, propagation over long distances will necessarily show their effects. Finally, the effects of stratification remains to be ascertained for realistic depths, ranges, and sound speed gradients, although results presented recently by Cotaras *et al* seem to show that the effect of inhomogeneity on nonlinear propagation is small.

The work of Cotaras *et al* is important to the subject of this review, as these investigators conducted extensive numerical computations with the specific purpose of assessing the importance of nonlinearities in the propagation of two different types of model pulses. The first, generated at a depth of 300 m, consisted of a shock wave followed by an exponential decay. The second pulse included the first bubble peak and was generated at a depth of 4300 m. In both cases, the effects of nonlinearity, dispersion, and attenuation were estimated by numerically allowing the pulses to travel long distances. The numerical algorithm used was based on geometrical acoustics, and included the effects of nonlinearity, attenuation and dispersion.

Their conclusions regarding long range propagation seem to indicate that nonlinearity is more important at moderate ranges than our crude estimates show. For the exponentially decaying pulse they present pressure profiles at various distances which show significant lengthening taking place at moderate absolute ranges. In the second type of pulse, they state that beyond a certain distance which depends on frequency and source strengths, nonlinear effects can be disregarded. Nonlinearity was found to have no effects below 4000 Hz.

While these conclusions are based on possibly the most complete study ever made, they are nevertheless open to question because of the values used for some of the assumed initial conditions. These are as follows:

1. The reference, or initial, value of R was taken to be 1.31 ft for the 1.8 lb charges and 3.61 ft for the 50 lb charges. These distances probably give values of R_o/L^o_{pp} which are too small for the theory to be applicable.

Incidentally, a 50 lb TNT charge has a lateral dimension of about 1 ft, and *this not too small when compared to 3.61 ft.*

2. The initial value of $P_o^o/\rho_o c_o^2$ was taken equal to 0.06 for both charges. While this is probably within the limits of the nonlinear theory, it corresponds to a value of $W^{1/3}/R$ equal to 0.92 for the 1.8 lb charge and equal to 1.02 for the 50 lb charge. These values are beyond the upper range of validity of the fit ($W^{1/3}/R \leq 0.85$). This is relevant because the fit was used to obtain the charge weight needed to produce the assumed values of P_o at the desired initial ranges.

3. The effective value of our quantity τ used in the calculations was unity. As stated earlier, this value is perhaps unrealistically high.

OTHER EFFECTS CONSIDERED

In addition to the propagation effects considered above, there may be other effects that may play a role in scattering experiments as a result of the interaction of a shock wave and a free surface. One such effect is the possible nonlinear behavior of the reflection coefficient. Other possibilities include cavitation and surface motion induced by the pulses reaching the surface. Cavitation may create extensive bubbly regions near the surface which may affect the transmission of the surface-scattered signal. Surface motion, on the other hand, can radiate sound waves which may contaminate the signal.

As it turns out, none of these effects is important for the pulses used in the scattering experiments under consideration, as the charges used were considerably smaller than those required for these effects to be significant. These effects are mentioned here only because they were considered during the review.

Both cavitation and surface motion effects were given considerable attention in the forties, and the literature contains many papers on them.

Earlier papers by Arons contain schematic diagrams of the cavitation region below an air-water interface, due to an underwater explosion. Similar figures as well as a discussion of their effect on the backscattered signal may be found in a 1979 paper by Gaspin *et al.* The discussion below is mainly intended to provide a rough estimate of the charge weights needed to initiate cavitation.

The occurrence of cavitation may be considered in terms of plane waves reaching the surface, and some aspects of the reflection of plane shock waves have been treated in that manner by Wentzell *et al.* For our purposes, the spherical geometry is just as convenient, provided that the angles of incidence are not too close to the horizontal, because at such angles, a lateral wave appears which would invalidate the arguments presented in Appendix C. There, it is shown that due to the interaction of an incident and a reflected spherical wave, a region of considerable acoustic underpressure, that is, a region where the total pressure may be considerably below that of the ambient, is created at a small distance h below the surface. Provided that the charges are not too near the surface, that distance may be expressed as

$$h(\theta)/d = (c_o T_{min}/2d)\{1-c_o T_{min}/2d\cos\theta\}^{-1}$$

where T_{min} is the time between the pulse's front and the point of minimum pressure behind it, θ is the slant angle shown in Fig. C1, and d is the charge depth. Except for very small grazing angles, this gives a depth of a few meters below the surface. For example, an 8 lb charge of TNT detonated at a depth of 500 m, will produce the lowest pressure at about 4 m below the surface. As stated earlier, if this pressure is lower than the hydrostatic, then cavitation would be produced somewhere below the surface that could result in a layer of bubbly liquid. Such a layer could affect the backscattering strength significantly. The place where this is most likely to occur is directly above the charge. Thus for $\theta=0$ and at $h=h_o$, the total pressure is

$$P_{total}(h_o,\theta=0)= P_h + P_{min}(R_3,\theta^*=0) - P_0(R_1,\theta^*=0)$$

where P_h is the hydrostatic pressure, P_{min} is the minimum negative pressure in the incident pulse, and P_0 is the peak pressure in the reflected pulse. P_{min} and P_o may be computed using the experimental fits. Thus, assuming that cavitation occurs for $P_{total} \approx 0$, we must have

$$K_1 Z^{1/2} (w^{1/3}/R_3) + K_2 (W^{1/3}/R_1)^{1.13} \geq P_h$$

where the constants K_1 and K_2 are given by the fits of the minimum and peak pressure fits, respectively. The problem then reduces to finding the smallest values of $W^{1/3}/R_1$ (or those of $W^{1/3}/R_3$) that satisfy this approximate inequality. This can be done numerically with the aid of Eq. (C1) for R_1. For a charge detonated at a depth of 500 m, this procedure shows that the minimum charge weight would produce cavitation directly above the charge, at about 4 meters below the surface, is about 22 lb, or considerably larger than those actually used. Of course, as the burst depth is decreased, the charge weight required for cavitation is also decreased.

The second effect mentioned is the the possible re-radiation over long periods of time, by points on the surface due to oscillations induced there by the incoming pulse. The effect could be of some importance for strong explosions, but for the typical charges used in scattering experiments, it is negligible, as the maximum velocities induced then at the surface is a few cm/sec. These velocities are too low to produce any noticeable signals at the location of the receiver.

Both of the above results depend on the correctness of the pressure release boundary condition at the air-water interface. This is based on linear theory which might not be applicable to finite-amplitude pulses, specially in view of a recent report by Temperly which appears to indicate that, even in ideal situations, the reflection coefficient at such an interface differs from the value predicted by linear theory. Because of these reasons, we considered the reflection of a pulse at an interface between two fluid media, one of which (air in this case) might respond nonlinearly to a pulse propagating as a linear signal in the other. However,

as shown in Appendix D, the departures of the reflection coefficient α_r from unity are insignificant for typical incident pressure amplitudes, so that one can safely neglect nonlinear effects on normal incidence.

CONCLUSIONS

This review has considered the propagation characteristics over moderate ranges of pressure pulses due to small underwater explosions, such as those used in scattering experiments. We have concentrated our attention on the peak-pressure region, as it is here that nonlinear effects are likely to appear. Another reason for doing this is that experimental data show that other important portions of the pulse, such as the pressure minimum behind the leading shock, or the first bubble maximum decay is inversely proportional to the distance to the charge. That is, their decay is linear. We have found that there must be ranges during the propagation where nonlinear decay laws give values that are comparable to those predicted by the experimental fit. These ranges are found for moderate charges or at short distances from small charges. On the whole, it is concluded that nonlinear effects must be negligible for small charges. By implication, it is found that except for those ranges (as well as closer ones, where the front shock is too strong to be considered as a weak shock wave), the decay of the pressure peak in such pulses should be as predicted by linear acoustic theory. As this is not found experimentally, it is suggested that at least two effects were present in the experiments that are not represented by the $(W^{1/3}/R)^{1.13}$ fit. These are a possible effect of burst-depth on peak pressure, and a possible effect, due to horizontal stratification, on the amplitude of propagation along the vertical.

We have also found that realistic choices of the parameter τ appearing in the nonlinear theory produces good agreement between theory and experiment. This lends substantial support to the idea that the 1.13 exponent observed experimentally could indeed be due to nonlinear effects. However, as the nonlinear theory does not predict the same trend at long ranges, the 1.13 exponent found there remains to be explained.

RECOMMENDATIONS

The following aspects of the problem have not been considered in the past but may play a role in determining the propagation characteristics of the pressure pulse.

1. The effects of curvature of the wavefront on nonlinear decay. Current theories use an *ad-hoc* procedure, where a theory rigorously derived for plane waves is adapted to spherical waves. There is no theoretical development for small distances. Information provided by a spherical theory could be important because nonlinear predictions at subsequent ranges, use initial conditions close to the charge.

2. The possible reflection/amplification of vertically-propagating nonlinear waves due to horizontal stratification should be studied for profiles and ranges of actual interest. These effects may affect the empirical fit for the peak-pressure, particularly when large vertical ranges are considered.

3. The limitations of weak shock theory should be considered, in the same manner as they have been in gases. Thus, the basic restriction is that the quantity $P^o_o/\rho_o c_o^{\,2} = u_o/c_o$ be small. However, the value of this quantity in water, which results in errors of, say, 10% is not known. Such a value could properly delimit the ranges were the theory applies.

4. The effects of dissipation in the main part of the wave should be studied in some detail, at least for linear pulses.

5. Although not considered in this report, the propagation of acoustic pulses through liquids containing bubbly layers should be given considerable attention. Such layers are likely to exist below the sea surface as a result of various effects such as water waves breaking and entraining air into the ocean. In order to study these aspects, the interaction of sound pulses and single bubbles must be considered because that interaction determines, in part, the acoustic characteristics of a bubbly layer. Further, they are likely to produce considerable nonlinear effects.

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Appendix A

PRODUCTION OF SOUND PULSE DUE TO EXPLOSION

As stated in the introduction, the pressure pulse is due to the motion of the surface of the explosion bubble. This motion is oscillatory in time, but the oscillations are not linear nor periodic, as might be expected from the very large initial pressure differences. There has been a great deal of work on this problem, and much of it has been reported in the literature. It turns out that the motion of the bubble can be computed fairly accurately by assuming that the fluid around it is incompressible. See, for example, the text by Batchelor. The time history of the position of the surface in a bubble after an underwater explosion is depicted in the figure below, reproduced from a 1943 paper by Kennard. It will be used to guide our discussion of the waves emitted by the bubble.

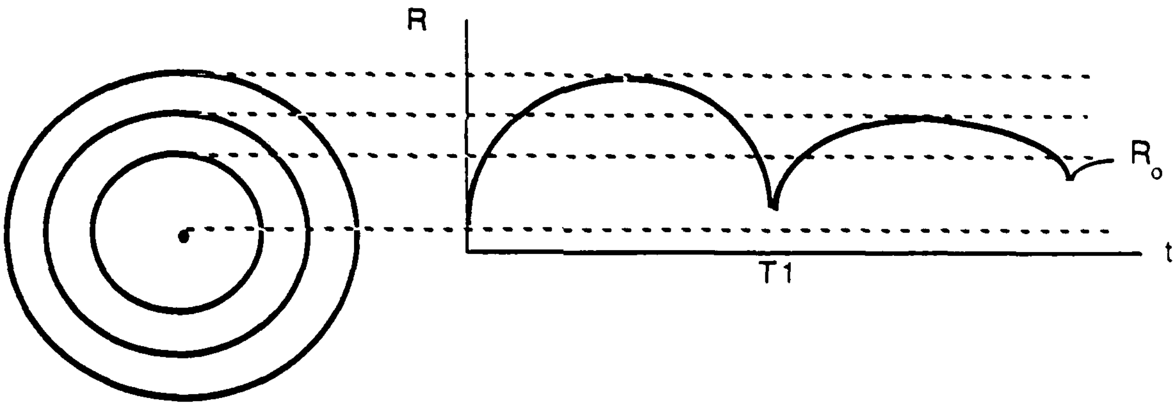

Fig. A1. Radial motion of gas bubble. R_0 denotes the radial position when the pressure inside the bubble equals that outside.

Generally speaking, two kinds of waves are produced at the moving interface separating the high pressure gases from the water: outgoing and incoming waves. Initially, the leading portions of each of these waves must necessarily be compressive for the waves sent into the water, and expansive for those sent into the bubble. If the pressure differences across the interface were small, one could use linear acoustic theory to analyze the waves produced by the sudden expansion of a spherical region,

initially at a pressure slightly larger than its surroundings. That theory shows that the sudden expansion creates a wave having a sharp front and a sharp tail, as depicted below at (a) some instant, and (b) at some location in space.

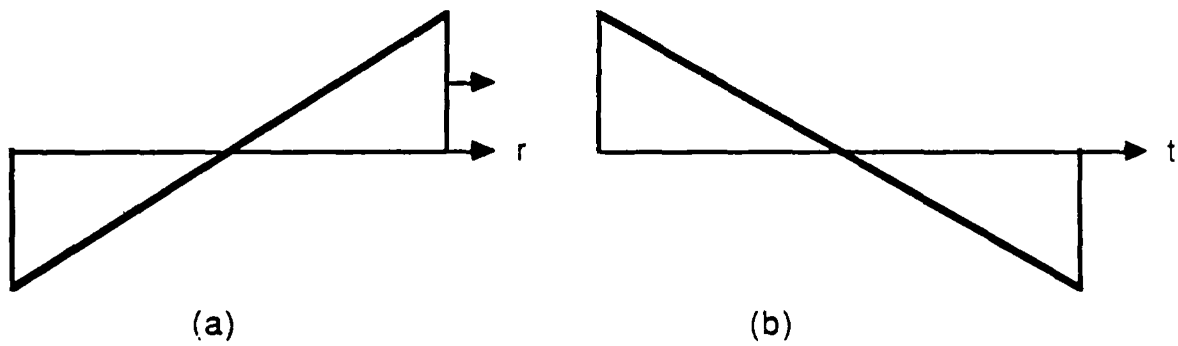

Fig. A2. Pressure waves produced by sudden expansion of pressurized region. (a) Profile at a fixed time. (b) Profile at fixed location

The negative acoustic pressures shown in the figures are due to the radially incoming expansion wave created at the interface, which at the times selected for the figures, has already been reflected at the origin. Now, in an actual explosion, the pressure differences are much too large for this model to apply to the expansion of the high pressure bubble. However, the expansion takes place at a considerably lower velocity than the velocity of the emitted waves. We may therefore visualize the actual expansion as taking place in a large number of very small steps, each producing two wavelets, one outgoing and one incoming, which travel behind those produced by the previous step. Consider first the wavelets sent outside the bubble. Their amplitude is proportional to the square of the instantaneous radius of the bubble and to the acceleration of the surface . As the radius is increasing and the acceleration is decreasing continuously during the expansion stages, the amplitude of each subsequent wavelet is roughly comparable with that of the first. However, the propagation velocity of each is larger than that of its predecessor. One of the results of this is that the leading front of the wave is reinforced by each of these wavelets. Therefore, the wavefront travelling into the water quickly becomes a rather strong shock wave, whose speed is considerably larger than that of the ambient speed c_0, ahead of the shock.

Consider now the expansion wavelets. These can be divided into two kinds: those produced before the first wave is reflected from the origin, and those produced afterwards, Because of the small size of typical charges, the travel time of the first expansion wave before reflection is very small compared with the time required by the bubble to expand to its maximum radius. Therefore, most of the wavelets produced during this first phase of the bubble motion are produced after the first wave is reflected from the origin. They therefore continue to be produced at longer distances from the origin, travelling towards it, then being reflected, and then continuing to travel behind the reflection of the first produced wave. One result of this is that the tail end of the wave is considerably elongated, relative to that of the first wave. Another is that the sharp tail is smoothed out somewhat. Hence, near the time the bubble has expanded to its maximum radius, the pressure profile at some distance from it, has the following shape

Fig. A3. Leading portion of outgoing wave.

At this stage, the bubble has reached its maximum radius. The motion does not end here because at this time, the pressure in the bubble has become smaller than the hydrostatic pressure around it. This has happened because during the expanding motion of the bubble, the water in its vicinity has acquired an outward motion, which does not cease the moment the pressure inside the bubble becomes equal to that outside. However, when the bubble has reached its maximum radius, the external pressure has become larger than that inside. It therefore begins to push the bubble in. The moment the bubble reverses direction, it begins to send outside wavelets whose fronts are expansive, and their tails are compressive. These wavelets join with the main pulse, producing a slow increase from the negative overpressure where it joins. The inward, radial motion of the

bubble continues for sometime, with the bubble now overshooting its equilibrium position once more, this time on its way to smaller radius. As the mass of the gases in the bubble is not large, and its temperature by now does not differ much from the ambient, the water is able to compress the bubble to a very small radius. Nevertheless, the pressure inside the bubble can become quite high, and results, when the bubble rebounds, in the emission of a second pressure pulse, called the first bubble pulse. The amplitude of this secondary pulse is smaller than that of the primary shock wave, but it can also have substantial amplitudes. Of course, the motion does not stop here, as the bubble can perform several of these oscillations. In each subsequent oscillation, the pulse amplitudes and bubble periods decrease. The combined pressure profile observed at some distance from the explosion can contain a few of these bubble pulses, as depicted schematically in Fig. 2. Also shown in the figure are the relative size of the bubble at the times when the corresponding pulse features were created.

Appendix B.

EXPERIMENTAL CORRELATIONS FOR PULSE PARAMETERS. TNT CHARGES*

All parameters are fitted by equations of the form $Y = KZ_o^{\alpha}(W^{1/3}/R)^{\beta}$

Parameter	Units	K	α	β	Limits of variables	
P_0	lb/in^2	20800	0	1.13	$7 \times 10^{-5} < W^{1/3}/R < 0.85$+	(B1)
P_{min}	lb/in^2	-77	1/3	-1.0	$14,000 \geq Z_o \geq 4500$	(B2)
P_1	lb/in	3300	0	-1.0	$4,000 \geq Z_o \geq 500$	(B3)
		875	1/6	-1.00	$14,000 \geq Z_o \geq 4500$	(B4)
$T_{pp}W^{-1/3}$	$sec/lb^{1/3}$	0.555	-5/6	0	$4,500 \geq Z_o \geq 500$	(B5)
		0.014	-2/5	0	$22,500 \geq Z_o \geq 4500$	(B6)
$T_1W^{-1/3}$	$sec/lb^{1/3}$	4.34	-5/6	0	$14,000 \geq Z_o \geq 4500$	(B7)

*Adapted from Slifko, 1967.

+Includes high range data of Coles et al.

Appendix C.

SPHERICAL PULSE FROM REFLECTION AT AN AIR-WATER INTERFACE

In this appendix we consider the reflection of an incident spherical pulse reaching the air-water surface from the water side, as depicted in the figure below. This shows a pulse produced at some depth reaching the surface from below, and its mirror image, an expansion wave of the same amplitude,

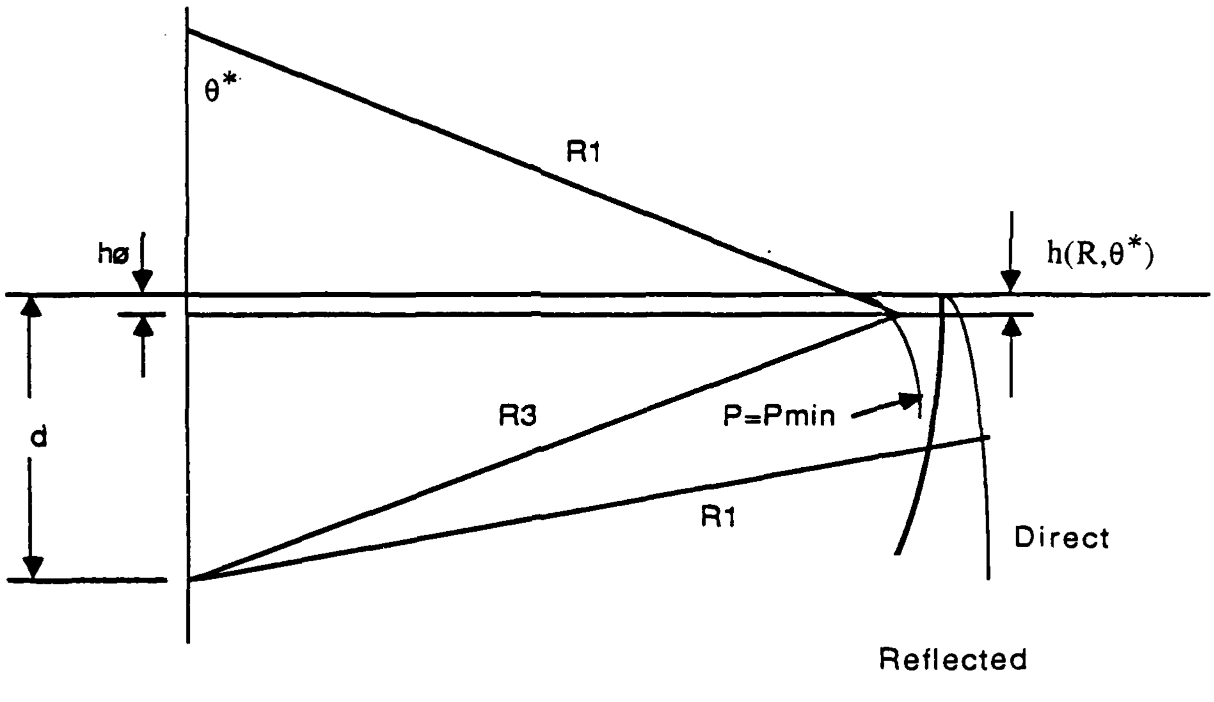

Fig. C1 Reflection at air-water interface

produced at the same distance above the surface. The combination insures that the pressure release boundary condition is satisfied everywhere on the surface. Now, as the expansion wave moves into the liquid, it interacts with the pressure profile behind the incident pulse. The loci where the

expansion wavefront meets the point of minimum pressure, P_{min}, in the incident wave, determines the lowest possible underpressure in the liquid. If this underpressure reaches a magnitude equal to the hydrostatic pressure at that location, cavitation is likely to occur because real sea water, containing as it does, dust particles having small pockets of trapped air, cannot sustain negative pressures. Cavitation can occur earlier in the interaction between the two waves, but we concentrate on that location as it yields the lowest possible pressure. Now, as denoted in the figure, R_1 represents the distance from the image charge to the front of the expansion wave, and R_3 the distance from the actual charge to the location of the minimum pressure behind the incident pulse. That front is at a distance $R_1 = R_1 + c_o T_{min}$ from the charge. From the geometry of the problem it follows that

$$R_3 = \{R_1^2 - 4dR_1\cos\theta^* + 4\,d^2\}^{1/2}$$

Because the point of minimum pressure is located very close to the incident wavefront, it follows that the distance from the surface where the intersection occurs should be small compared to either R or to d. Thus, it follows that $h/d = (R_1\cos\theta^*/d) - 1 << 1$. This can be used in the equation for R_3 above to yield

$$R_3 \equiv R_1\{1 - [(R_1\cos\theta^*/d) - 1]/2(R_1/2d^2)\}$$

In addition, $R_1 = c_o t$ and $R_3 = c_o(t - T_{min})$, where T_{min} is the time between the wavefront and the location of minimum pressure. Combining these results, we obtain

$$R_1 = d\,\{\cos\theta^* - c_o T_{min}\min/2d\}^{-1} \qquad \text{(C1)}$$

This gives the distance from the image charge. The distance to the surface for any angle θ is simply $R_s = d/\cos\theta$. Thus, the distance below the surface where the minimum pressure occurs is $h = (R - R_s)\cos\theta$, or

$$h(\theta) = d\,(c_o T_{min}/2d)\{1 - c_o T_{min}/2d\cos\theta\}^{-1}$$

as given on page 28.

Appendix D

REFLECTION COEFFICIENT AT AIR-WATER INTERFACE

In this appendix we consider the reflection of a normal, plane wave from a water-air interface. The incident pulse reaches the interface from the water side, where it travels as a linear acoustic wave. Because of the interface, a fraction of the acoustic energy is reflected back into the water, and part is transmitted into the air. The reflected pulse is also well described by linear acoustics. In the linear approximation, the transmitted pulse also behaves linearly, so that the reflection coefficient is given by

$$\alpha_r = \left| \frac{1-Z_{21}}{1+Z_{21}} \right|^2$$

where $Z_{21}=\rho_2 c_2/\rho_1 c_1$. Although the smallness of this ratio dictates that the transmitted wave should have a rather small amplitude, it is in principle possible that the small compressibility of the air, $\rho_2 c_2^2$, relative to that of the water, can result in a value of $P_2(0)/\rho_2 c_2^2$ that is too large for linear theory to apply.

To study that possibility, we consider a plane wave whose velocity profile is given by $f(x-c_1 t)$ traveling towards the interface. The particle velocity in the reflected wave is given by $g(x+c_1 t)$. At the interface, the pressures and velocities on either side match, so that

$$p_o + \rho_1 c_1 [f(-c_1 t) - g(c_1 t)] = P_2(0) \qquad \text{(D1)}$$

$$\rho_1 c_1 [f(-c_1 t) + g(c_1 t)] = u_2(0) \qquad \text{(D2)}$$

where p_o is the ambient pressure at the interface, and $P_2(0)$ and $u_2(0)$ are, respectively, the total pressure and velocity on the air side. Now, for simple waves in a perfect gas, the pressure is related to the velocity by means of

$$P_2(0) = \{[1-(\gamma-1)u_2(0)/c_2]^{[2\gamma/(\gamma-1)]} -1\} \qquad \text{(D3)}$$

We are interested in the ratio g(0,t)/f(0,t), as it is this ratio that determines the value of the reflection coefficient. Combining (D1)-(D3), and introducing

$$\beta = 2[1 + g(0,t)/f(0,t)]/(1+Z_{21})$$

we obtain

$$\beta = \{2 -(c_2/c_1)(Z_{21}/\gamma\varepsilon)[1-(\gamma-1)(c_2/c_1)\beta\varepsilon/(1+Z_{21})]^{[2\gamma/(\gamma-1)]} -1\}$$

where ε is given by $\varepsilon = P_1(0)/\rho_1 c_1^2$. In terms of β , the reflection coefficient is given by

$$\alpha_r = \left|\frac{2\beta-1-Z_{21}}{1+Z_{21}}\right|^2 \qquad \text{(D4)}$$

The linear case is obtained in the limit $\varepsilon \ll 1$. This yields $\beta = 1$, so that Eq. (D1) is obtained from (D4). The next order approximation for β, valid to order ε^2 and small Z_{21} gives

$$\beta = \frac{1}{1+2(\gamma-1)Z_{21}(c_1/c_2)\varepsilon}$$

which shows that $\beta<1$, so that α_r is less than one. However, the differences are insignificant. Thus, for $\varepsilon = 0.1$, β differs from unity by less than 1/10 of 1%.

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